

# Package ‘countDM’

July 22, 2025

**Type** Package

**Title** Estimation of Count Data Models

**Version** 0.1.0

**Author** Muhammad Imran [aut, cre],  
M.H. Tahir [aut],  
Saima Shakoor [aut]

**Maintainer** Muhammad Imran <imranshakoor84@yahoo.com>

**Depends** R (>= 4.0)

**Imports** lamW, stats, numbers, maxLik, miscTools

## Description

The maximum likelihood estimation (MLE) of the count data models along with standard error of the estimates and Akaike information model selection criterion are provided. The functions allow to compute the MLE for the following distributions such as the Bell distribution, the Borel distribution, the Poisson distribution, zero inflated Bell distribution, zero inflated Bell Touchard distribution, zero inflated Poisson distribution, zero one inflated Bell distribution and zero one inflated Poisson distribution. Moreover, the probability mass function (PMF), distribution function (CDF), quantile function (QF) and random numbers generation of the Bell Touchard and zero inflated Bell Touchard distribution are also provided.

**License** GPL (>= 2)

**Encoding** UTF-8

**RoxygenNote** 7.2.3

**NeedsCompilation** no

**Repository** CRAN

**Date/Publication** 2023-05-25 20:10:02 UTC

## Contents

countDM-package . . . . .	2
Bell . . . . .	3
Bell Touchard . . . . .	4
Borel . . . . .	6

Criminal acts . . . . .	7
Poisson . . . . .	8
Stillbirths . . . . .	9
Touchard polynomials . . . . .	10
Zero inflated Bell . . . . .	11
Zero inflated Bell Touchard . . . . .	12
Zero inflated Poisson . . . . .	14
Zero one inflated Bell . . . . .	15
Zero one inflated Poisson . . . . .	16

<b>Index</b>	<b>18</b>
--------------	-----------

---

countDM-package	<i>Estimation of Count Data Models</i>
-----------------	--

---

## Description

It gives the maximum likelihood estimates and the corresponding estimate's standard error. It also provides the Akaike information model section criterion. With the help of these functions, the MLE can be calculated for a variety of distributions, including the Borel distribution, the Poisson distribution, zero inflated Bell distribution, zero inflated Bell Touchard distribution, zero inflated Poisson distribution, zero one inflated Bell distribution and zero one inflated Poisson distribution. Moreover, the probability mass function, distribution function, quantile function and random numbers generation of the Bell Touchard and zero inflated Bell Touchard distribution are also provided.

## Details

Package: countDM  
 Type: Package  
 Version: 0.1.0  
 Date: 2023-05-24  
 License: GPL(>=2)

## Maintainers

Muhammad Imran <imranshako0r84@yahoo.com>

## Author(s)

Muhammad Imran <imranshako0r84@yahoo.com>, M.H. Tahir <mht@iub.edu.pk> and Saima Shako0r <saimashako0r500@gmail.com>.

**Description**

Evaluates the maximum likelihood estimate of the Bell distribution. The PMF of the Bell distribution is as follows:

$$f(X = x | \theta) = \frac{\theta^x e^{e^\theta + 1} B_x}{x!}; \quad x = 0, 1, 2, \dots,$$

where  $\theta > 0$  denotes the Bell parameter and  $B_x$  is the Bell number and it is given by

$$B_n = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!}.$$

The Bell number  $B_n$  in the above equation is the  $n$ th moment of the Poisson distribution with parameter equal to 1.

**Usage**

```
bell_mle (x)
mle.bell (x, theta)
```

**Arguments**

x	A vector of (non-negative integer) discrete values.
theta	A vector of (non-negative integer) values.

**Details**

The function allows to estimate the unknown parameter of the Bell distribution with loglik value using a Newton-Raphson algorithm.

**Value**

bell\_mle gives the maximum likelihood estimate of parameter theta. loglik gives value of the maximised log-likelihood. The mle.bell gives the maximum likelihood estimate with standard error and AIC,

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Castellares, F., Ferrari, S. L., & Lemonte, A. J. (2018). On the Bell distribution and its associated regression model for count data. *Applied Mathematical Modelling*, 56, 172-185.

**See Also**

[mle\\_borel](#), [mle\\_poisson](#)

**Examples**

```
x <- data_sbirth
bell_mle(x)
mle.bell(x, 1.2)
```

---

Bell Touchard

*PMF, CDF, QF, random generation and parameters estimation based on the Bell Touchard distribution*

---

**Description**

Evaluates the PMF, CDF, QF, random generation and MLE based on the Bell Touchard distribution. The PMF of the Bell Touchard distribution is as follows:

$$f(X = x | \lambda, \theta) = \exp\{\theta [1 - e^{-\lambda}]\} \frac{\lambda^x T_x(\theta)}{x!}; \quad x = 0, 1, 2, \dots,$$

where  $\lambda > 0$  and  $\theta > 0$  are the two parameters and  $T_x$  are the Touchard polynomials, it is given by

$$T_n = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!}.$$

It is important to note that when the parameter  $\theta = 1$ , the Bell Touchard distribution reduces to Bell distribution.

**Usage**

```
dbellt(x, lambda, theta, log = FALSE)
pbellt(q, lambda, theta, lower.tail = TRUE, log.p = FALSE)
qbellt(p, lambda, theta, lower.tail = TRUE, log.p = FALSE)
rbellt(n, lambda, theta)
mle_bt(x, lambda, theta)
```

**Arguments**

x	A vector of (non-negative integer) discrete values.
lambda	A vector of (non-negative integer) values, $\lambda > 0$ .
theta	A vector of (non-negative integer) values, $\theta > 0$ .
q	A probability or a vector of probabilities.
p	A probability or a vector of probabilities.
n	A randomly generated values.
lower.tail	logical; if TRUE (default), probabilities will be $P[X \leq x]$ , otherwise, $P[X > x]$ .
log	logical; if TRUE, probabilities p are given as $\log(p)$ .
log.p	logical; if TRUE, probabilities p are given as $\log(p)$ .

## Details

Recently Castellares et al. (2020) proposed a two parameter discrete Bell Touchard distribution that overcomes the issue of over-dispersion (variance larger than mean). Often, we experience the over-dispersed data in practice, where the Poisson model may not be suitable because its variance is restricted to be equal to mean. The PMF of the Bell Touchard distribution is simple and tractable in order to find probabilities as well as several properties of the distribution.

## Value

`dbellt` gives the (log) probability function. `pbellt` gives the (log) distribution function. `qbellt` gives the quantile function. `rbellt` generates random values. `mle_bt` gives the maximum likelihood estimates with standard error of the estimates and model selection measure, the Akaike information criterion (AIC).

## Author(s)

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

## References

Castellares, F., Lemonte, A. J., & Moreno–Arenas, G. (2020). On the two-parameter Bell–Touchard discrete distribution. *Communications in Statistics-Theory and Methods*, 49(19), 4834-4852.

Castellares, F., Ferrari, S. L., & Lemonte, A. J. (2018). On the Bell distribution and its associated regression model for count data. *Applied Mathematical Modelling*, 56, 172-185.

## See Also

[mle\\_borel](#), [mle\\_poisson](#), [bell\\_mle](#)

## Examples

```
x<-2
dbellt(x, 2, 2)
pbellt(0.2, 2, 2)
pbellt(0.2, 2, 2)
rbellt(10, 2, 1)
x <- data_sbirth
mle_bt(x, 0.12, 0.2)

# Plot of PMF of the Bell Touchard distribution for selected parameteric values.
# The Bell Touchard distribution reduces to the Bell distribution for fixing theta=1.
x <- 0:50
lambda <- 2
theta <- 1.5

plot(dbellt(x, lambda, theta), type = "h", col="red",lwd = 3,
     main = "Bell Touchard distribution",
     ylab = "P(X = x)", xlab = "Number of events")
```

```
legend("topright",c(expression(lambda==2~theta==1.5)),lty=1, col="red", lwd=2,cex=1.0)
```

Borel

*MLE of the Borel distribution***Description**

Evaluates the MLE of the Borel distribution. It is defined by the following PMF:

$$f(X = x | \alpha) = \frac{(\alpha x)^{x-1} e^{-\alpha x}}{x!}; \quad x = 1, 2, \dots,$$

where the parameter  $\alpha \in (0, 1)$ .

**Usage**

```
mle_borel (x, alpha)
```

**Arguments**

`x`                    A vector of (non-negative integer) discrete values.  
`alpha`                A vector of (non-negative integer) values,  $\alpha \in (0, 1)$ .

**Details**

The function allows to estimate the unknown parameter of the Borel distribution with standard error of the estimate and model selection measure, the Akaike information criterion (AIC).

**Value**

`mle_borel` gives the MLE along with standard error of the estimate and model selection measure AIC.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Tanner, J. C. (1961). A derivation of the Borel distribution. *Biometrika*, 48(1/2), 222-224.

**See Also**

[mle.bell](#), [mle\\_poisson](#)

**Examples**

```
x <- c(rep(1,48), rep(2,20), rep(3,7), rep(4,5),rep(5,2),rep(6,6))
mle_borel (x, 0.8)
```

---

Criminal acts	<i>The crime sociology consisting a sample of 4301 people with deviating behavior</i>
---------------	---

---

**Description**

The data set from crime sociology consisting a sample of 4301 people with deviating behavior.

**Usage**

```
data_criminal
```

**Arguments**

```
data_criminal  A vector of (non-negative integer) count values.
```

**Details**

The data set from crime sociology consisting a sample of 4301 people with deviating behavior. Recently, it is used by Zhang et al. (2016), fitted the zero one inflated Poisson distribution.

**Value**

data\_criminal gives the crime sociology consisting a sample of 4301 people with deviating behavior.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Zhang, C., Tian, G. L., & Ng, K. W. (2016). Properties of the zero-and-one inflated Poisson distribution and likelihood-based inference methods. *Statistics and its interface*, 9(1), 11-32.

**See Also**

[data\\_sbirth](#)

**Examples**

```
x <- data_criminal
table (x)
```

**Description**

Evaluates the MLE of the Poisson distribution. The PMF of the Poisson distribution is as follows:

$$f(X = x | \theta) = \frac{\theta^x e^{-\theta}}{x!}; \quad x = 0, 1, 2, \dots,$$

where parameter  $\theta > 0$  and it is equal to the expected or mean value of  $X$  and also to its variance.

**Usage**

```
mle_poisson (x, theta)
```

**Arguments**

x	A vector of (non-negative integer) discrete values.
theta	A vector of (non-negative integer) values, $\theta > 0$ .

**Details**

The function allows to estimate the unknown parameter of Poisson distribution with standard error of the estimate and model selection measure, the Akaike information criterion (AIC).

**Value**

mle\_poisson gives the MLE along with standard error of the estimate and model selection measure AIC.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshakoor84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Frank A. Haight (1967). Handbook of the Poisson Distribution. New York: John Wiley & Sons.

**See Also**

[mle.bell](#), [mle.borel](#)

**Examples**

```
x <- data_sbirth
mle_poisson (x, 0.2)
```



---

Stillbirths

*Still births of New Zealand white rabbits*

---

**Description**

The number of stillbirths of New Zealand white rabbits.

**Usage**

```
data_sbirth
```

**Arguments**

```
data_sbirth
```

 A vector of (non-negative integer) count values.**Details**

The data set consists of frequencies of still births in 402 litters of New Zealand white rabbits, originally used by Morgan et al. (2007).

**Value**

`data_sbirth` gives the frequencies of still births in 402 litters of New Zealand white rabbits.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Morgan, B. T., Palmer, K. J., & Ridout, M. S. (2007). Negative score test statistic. *The American Statistician*, 61(4), 285-288.

Alshkaki, R. S. A. (2016). On the zero-one inflated Poisson distribution. *Int J Stat Distrib Appl*, 2(4), 42-8.

Pudprommarat, C. (2020, March). Zero-one inflated negative binomial-Sushila distribution and its application. In *International Academic Multidisciplinary Research Conference in Rome 2020* (pp. 20-28).

**See Also**

[data\\_criminal](#)

**Examples**

```
x <- data_sbirth
table(x)
```

**Description**

The function allows to compute the Touchard polynomial. It is mathematically defined by

$$T_x(\theta) = \frac{1}{e^\theta} \sum_{k=0}^{\infty} \frac{k^x}{k!} \theta^k.$$

The first few Touchard polynomials are as follows:

$$\left\{ \begin{array}{l} T_0(\theta) = 1 \\ T_1(\theta) = \theta \\ T_2(\theta) = \theta^2 + \theta \\ T_3(\theta) = \theta^3 + 3\theta^2 + \theta \\ T_4(\theta) = \theta^4 + 6\theta^3 + 7\theta^2 + \theta. \end{array} \right. .$$

**Usage**

TP (x, theta)

**Arguments**

x                    A vector of (non-negative integer) discrete values.  
theta                A vector of (non-negative integer) values.

**Details**

The function allows to provide the Touchard polynomials.

**Value**

TP gives the Touchard polynomials after specifying parameteric value.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshakoor84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Castellares, F., Lemonte, A. J., & Moreno–Arenas, G. (2020). On the two-parameter Bell–Touchard discrete distribution. *Communications in Statistics-Theory and Methods*, 49(19), 4834-4852.

**Examples**

TP (2,3)

---

 Zero inflated Bell      *MLE of the zero inflated Bell distribution*


---

### Description

Evaluates the MLE of the zero inflated Bell (ZIBELL) distribution. The PMF of the ZIBELL distribution is as follows:

$$f(X = x | \alpha, \lambda) = \begin{cases} \alpha + (1 - \alpha) \exp\{\theta [1 - e^\lambda]\}, & x = 0 \\ (1 - \alpha) \exp\{\theta [1 - e^\lambda]\} \frac{\lambda^x B_x}{x!}, & x = 1, 2, \dots, \end{cases}$$

where  $\alpha \in (0, 1)$ ,  $\lambda > 0$  and  $B_x$  are the Bell numbers and it is given by

$$B_n = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!}.$$

### Usage

```
mle_zibell(x, alpha, lambda)
```

### Arguments

x	A vector of (non-negative integer) values, discrete values.
lambda	A vector of (non-negative integer) values, $\lambda > 0$ .
alpha	A vector of (non-negative integer) values, $\alpha \in (0, 1)$ .

### Details

The function allows to estimate the unknown parameter of the ZIBELL distribution with standard error of the estimate and model selection measure, the Akaike information criterion (AIC).

### Value

mle\_zibell gives the MLE along with standard error of the estimate and model selection measure AIC.

### Author(s)

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshakoar84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

### References

Castellares, F., Ferrari, S. L., & Lemonte, A. J. (2018). On the Bell distribution and its associated regression model for count data. *Applied Mathematical Modelling*, 56, 172-185.

**See Also**[mle\\_zibellt](#)**Examples**

```
x <- data_sbirth
mle_zibell (x, 0.2, 1.5)
```

Zero inflated Bell Touchard

*MLE of the zero inflated Bell Touchard distribution***Description**

Evaluates the maximum likelihood estimate of the zero inflated Bell Touchard (ZIBELLT) distribution. The PMF of the ZIBELLT distribution is as follows:

$$f(X = x | p_i, \lambda, \theta) = \begin{cases} p_i + (1 - p_i) \exp \{ \theta [1 - e^\lambda] \}, & x = 0 \\ (1 - p_i) \exp \{ \theta [1 - e^\lambda] \} \frac{\lambda^x T_x(\theta)}{x!}, & x = 1, 2, \dots, \end{cases}$$

where  $p_i \in (0, 1)$ ,  $\lambda > 0$  and  $\theta > 0$   $T_x$  are the Touchard polynomials, it is given by

$$T_n = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!}.$$

It is important to note that when the parameter  $\theta = 1$ , the ZIBELLT distribution reduces to ZIBELL distribution. On the other side, when the parameter  $\theta = 1$  and  $p_i=0$ , the ZIBELLT distribution reduces to BELL distribution. So therefore, we can evaluate the PMF, CDF, QF and random numbers of the Bell and ZIBELL distribution by using the following functions.

**Usage**

```
dzibellt (x, lambda, theta, pi, log = FALSE)
pzibellt (q, lambda, theta, pi, lower.tail = TRUE, log.p = FALSE)
qzibellt (p, lambda, theta, pi, lower.tail = TRUE, log.p = FALSE)
rzibellt (n, lambda, theta, pi)
mle_zibellt (x, lambda, theta, pi)
```

**Arguments**

x	A vector of (non-negative integer) discrete values.
lambda	A vector of (non-negative integer) values, $\lambda > 0$ .
theta	A vector of (non-negative integer) values, $\theta > 0$ .
n	The number of random values generated under zero inflated Bell Touchard distribution.
pi	A vector of (non-negative integer) values, $p_i \in (0, 1)$ .

<code>q</code>	A vector of (non-negative integer) probabilities.
<code>p</code>	A vector of (non-negative integer) probabilities.
<code>lower.tail</code>	logical; if TRUE (default), probabilities will be $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>log</code>	logical; if TRUE, probabilities <code>p</code> are given as $\log(p)$ .
<code>log.p</code>	logical; if TRUE, probabilities <code>p</code> are given as $\log(p)$ .

### Details

Recently Castellares et al. (2020) proposed a two parameter discrete Bell Touchard distribution that overcomes the issue of over-dispersion (variance larger than mean). Often, we experience the over-dispersed data in practice, where the Poisson model may not be suitable because its variance is restricted to be equal to mean. The PMF of the Bell Touchard distribution is simple and tractible in order to find probabilities as well as several properties of the distribution. We extend it to the ZIBELLT distribution and evaluated by the following functions.

### Value

`dzibellt` gives the (log) probability function. `pzibellt` gives the (log) distribution function. `qzibellt` gives the quantile function. `rzibellt` generates random values. `mle_zibell` gives the maximum likelihood estimates with standard error of the estimates and model selection measure, the Akaike information criterion (AIC).

### Author(s)

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

### References

Castellares, F., Lemonte, A. J., & Moreno–Arenas, G. (2020). On the two-parameter Bell–Touchard discrete distribution. *Communications in Statistics-Theory and Methods*, 49(19), 4834-4852.

Castellares, F., Ferrari, S. L., & Lemonte, A. J. (2018). On the Bell distribution and its associated regression model for count data. *Applied Mathematical Modelling*, 56, 172-185.

### See Also

[mle\\_zibell](#)

### Examples

```
dzibellt (2, 0.12, 0.2,0.2)

x <- data_sbirth
mle_zibellt (x, 0.15,1.8,1.05)

# Plot of PMF of the ZIBELLT distribution for selected parameteric values.
# The ZIBELLT distribution reduces to the ZIBELL distribution for fixing theta=1.
```

```
x <- 0:30
lambda <- 1.2
theta <- 1.2
pi <- 0.01
plot(dzibellt(x, lambda, theta, pi), type = "h", col="red",lwd = 3,
     main = "Zero inflated Bell Touchard distribution",
     ylab = "P(X = x)", xlab = "Number of events")

legend("topright",c(expression(lambda==2~~theta==1.5~~pi==0.01)),
      lty=1, col="red", lwd=2,cex=1.0)
```

---

Zero inflated Poisson *MLE of the zero inflated Poisson distribution*

---

### Description

Evaluates the MLE of the zero inflated Poisson (ZIP) distribution. The PMF of the ZIP is as follows:

$$f(X = x | \alpha, \theta) = \begin{cases} \alpha + (1 - \alpha) e^{-\theta}, & x = 0 \\ (1 - \alpha) \frac{\theta^x e^{-\theta}}{x!}, & x = 1, 2, \dots, \end{cases}$$

where  $\alpha \in (0, 1)$  denotes the probability of extra zeros and  $\theta > 0$  is a Poisson parameter, which is also its mean and variance.

### Usage

```
mle_zip(x, alpha, theta)
```

### Arguments

x	A vector of (non-negative integer) discrete values.
theta	A vector of (non-negative integer) values, $\theta > 0$ .
alpha	A vector of (non-negative integer) values, $\alpha \in (0, 1)$ .

### Details

The function allows to estimate the unknown parameter of the ZIP distribution with standard error of the estimate and model selection measure, the Akaike information criterion (AIC).

### Value

mle\_zip gives the MLE along with standard error of the estimate and model selection measure AIC.

### Author(s)

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshakoor84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Beckett, S., Jee, J., Ncube, T., Pompilus, S., Washington, Q., Singh, A., & Pal, N. (2014). Zero-inflated Poisson (ZIP) distribution: Parameter estimation and applications to model data from natural calamities. *Involve, a Journal of Mathematics*, 7(6), 751-767.

**See Also**

[mle\\_zibell](#), [mle\\_zibellt](#)

**Examples**

```
x <- data_sbirth
mle_zip (x, 0.2, 1.5)
```

Zero one inflated Bell

*MLE of the zero one inflated Bell distribution*

**Description**

Evaluates the MLE of the zero one inflated Bell (ZOIBELL)distribution.

$$f(X = x | \alpha, \beta, \theta) = \begin{cases} \alpha + (1 - \alpha - \beta) \exp(1 - e^\theta), & x = 0 \\ \beta + (1 - \alpha - \beta) \theta \exp(1 - e^\theta), & x = 1 \\ (1 - \alpha - \beta) \exp(1 - e^\theta) \frac{\theta^x B_x}{x!}, & x = 2, 3, \dots, \end{cases}$$

where  $\theta > 0$ , the two parameters  $\alpha \in (0, 1)$  and  $\beta \in (0, 1)$ , respectively, denotes the unknown proportion for incorporating extra zeros and extra ones than those allowed by the traditional Bell distribution.

**Usage**

```
mle_zoibell (x, alpha, beta, theta)
```

**Arguments**

x	A vector of (non-negative integer) values, discrete values.
theta	A vector of (non-negative integer) values, $\theta > 0$ .
alpha	A vector of (non-negative integer) values, $\alpha \in (0, 1)$ .
beta	A vector of (non-negative integer) values, $\beta \in (0, 1)$ .

**Details**

Inflated models have become quite popular in the recent applied statistical literature. In many scientific studies, we often experience situations, the data consists of a large proportion of zeros and ones. Castellares et al. proposed a single-parameter discrete Bell distribution. We extended the Bell distribution into zero one inflated Bell distribution.

**Value**

mle\_zoibell gives the MLE along with standard error of the estimate and model selection measure AIC.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshakoor84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Castellares, F., Ferrari, S. L., & Lemonte, A. J. (2018). On the Bell distribution and its associated regression model for count data. *Applied Mathematical Modelling*, 56, 172-185.

**See Also**

[mle\\_zoip](#)

**Examples**

```
x <- data_sbirth
mle_zoibell(x, 0.1, 0.2, 0.2)
```

---

Zero one inflated Poisson

*MLE of the zero one inflated Poisson distribution*

---

**Description**

Evaluates the MLE of the zero one inflated Poisson (ZOIP) distribution.

$$f(X = x | \alpha, \beta, \theta) = \begin{cases} \alpha + (1 - \alpha - \beta) e^{-\theta}, & x = 0 \\ \beta + (1 - \alpha - \beta) \theta e^{-\theta}, & x = 1 \\ (1 - \alpha - \beta) e^{-\theta} \frac{\theta^x}{x!}, & x = 2, 3, \dots \end{cases}$$

where  $\theta > 0$ , the two parameters  $\alpha \in (0, 1)$  and  $\beta \in (0, 1)$ , respectively denotes the unknown proportion for incorporating extra zeros and extra ones than those allowed by the traditional Poisson distribution.

**Usage**

```
mle_zoip(x, alpha, beta, theta)
```



**Arguments**

x	A vector of (non-negative integer) values, discrete values.
alpha	A vector of (non-negative integer) values, $\alpha \in (0, 1)$ .
beta	A vector of (non-negative integer) values, $\beta \in (0, 1)$ .
theta	A vector of (non-negative integer) values, $\theta > 0$ .

**Details**

The function allows to estimate the unknown parameter of the ZOIP distribution with standard error of the estimate and model selection measure, the Akaike information criterion (AIC).

**Value**

mle\_zoip gives the MLE along with standard error of the estimate and model selection measure AIC.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

- Zhang, C., Tian, G. L., & Ng, K. W. (2016). Properties of the zero-and-one inflated Poisson distribution and likelihood-based inference methods. *Statistics and its interface*, 9(1), 11-32.
- Tang, Y., Liu, W., & Xu, A. (2017). Statistical inference for zero-and-one-inflated Poisson models. *Statistical Theory and Related Fields*, 1(2), 216-226.
- Alshkaki, R. S. A. (2016). On the zero-one inflated Poisson distribution. *Int J Stat Distrib Appl*, 2(4), 42-8.

**See Also**

[mle\\_zoibell](#)

**Examples**

```
x <- data_sbirth
mle_zoip(x, 0.2, 0.1, 0.5)
```

# Index

- \* **Count data models**
    - countDM-package, 2
  - \* **Discrete probability distributions**
    - countDM-package, 2
  - \* **Inflated models**
    - countDM-package, 2
  - \* **Overdispersion**
    - countDM-package, 2
- Bell, 3  
Bell Touchard, 4  
bell\_mle, 5  
bell\_mle (Bell), 3  
Borel, 6
- countDM (countDM-package), 2  
countDM-package, 2  
Criminal acts, 7
- data\_criminal, 9  
data\_criminal (Criminal acts), 7  
data\_sbirth, 7  
data\_sbirth (Stillbirths), 9  
dbellt (Bell Touchard), 4  
dzibellt (Zero inflated Bell Touchard), 12
- mle.bell, 6, 8  
mle.bell (Bell), 3  
mle\_borel, 4, 5, 8  
mle\_borel (Borel), 6  
mle\_bt (Bell Touchard), 4  
mle\_poisson, 4–6  
mle\_poisson (Poisson), 8  
mle\_zibell, 13, 15  
mle\_zibell (Zero inflated Bell), 11  
mle\_zibellt, 12, 15  
mle\_zibellt (Zero inflated Bell Touchard), 12  
mle\_zip (Zero inflated Poisson), 14
- mle\_zoibell, 17  
mle\_zoibell (Zero one inflated Bell), 15  
mle\_zoip, 16  
mle\_zoip (Zero one inflated Poisson), 16
- pbellt (Bell Touchard), 4  
Poisson, 8  
pzibellt (Zero inflated Bell Touchard), 12
- qbellt (Bell Touchard), 4  
qzibellt (Zero inflated Bell Touchard), 12
- rbellt (Bell Touchard), 4  
rzibellt (Zero inflated Bell Touchard), 12
- Stillbirths, 9
- Touchard polynomials, 10  
TP (Touchard polynomials), 10
- Zero inflated Bell, 11  
Zero inflated Bell Touchard, 12  
Zero inflated Poisson, 14  
Zero one inflated Bell, 15  
Zero one inflated Poisson, 16