Loci of Constant Hue and Chroma

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Introduction

The plots that Dorothy Nickerson made in [Newhall et al., 1943] and later in [Wyszecki and Stiles, 1982], are beautiful and a labor of love. But they often do not cover regions of high Chroma. And obviously one cannot zoom in to inspect problem areas in detail, which is often useful in development. The function `plotLociHC()` is designed to improve this situation.

```
library( munsellinterpol )
```

Zoom Out

Pick value 8/ for demonstration

```
par( omi=c(0,0,0,0), mai=c(0.41,0.7,0.6,0.2) )
plotLociHC( value=8 )
```

Value 8/

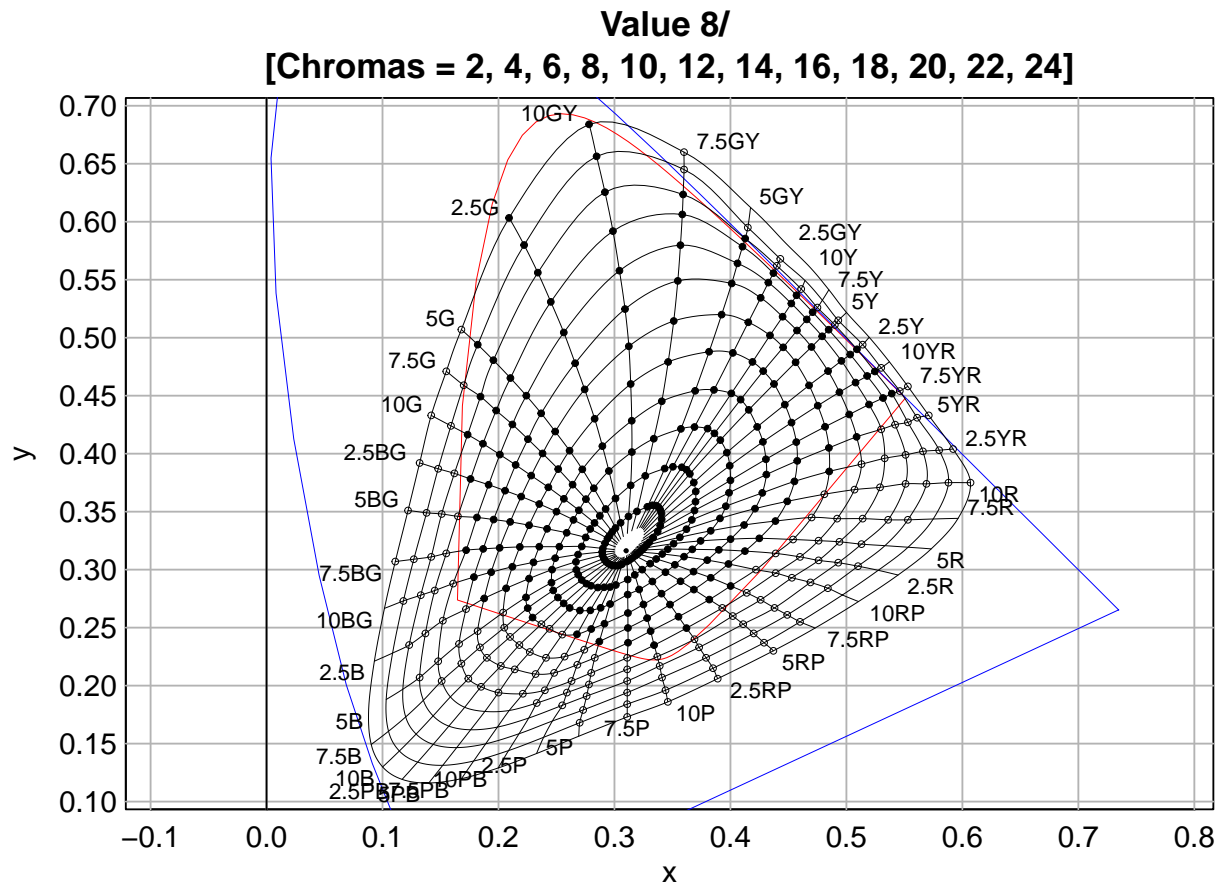**[Chromas = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24]**

Figure 1: Loci of Constant Hue and Chroma for Value 8/

The gridpoints marked with a black disk are published in [Newhall et al., 1943]. The gridpoints marked with a open circle are from `all.dat`, and almost certainly described in [Schleter et al., 1958], but not published. And the other gridpoints are pre-extrapolated by this package; see the **munsellinterpol** User Guide for details. The blue curve is the CIE inverted-U, and the red curve is the MacAdam limit for the given Value=8.

In `plotLociHC()` by default the largest "real" chroma is determined (C_{max}=24 for 8/), the complete oval at the chroma is computed, and the plot limits are set for that oval.

Plotting higher chroma in the 10GY area is easy:

```
par( omi=c(0,0,0,0), mai=c(0.41,0.7,0.6,0.2) )
plotLociHC( value=8, hue=seq(5,55,by=2.5), chroma=seq(2,34,by=2) )
```
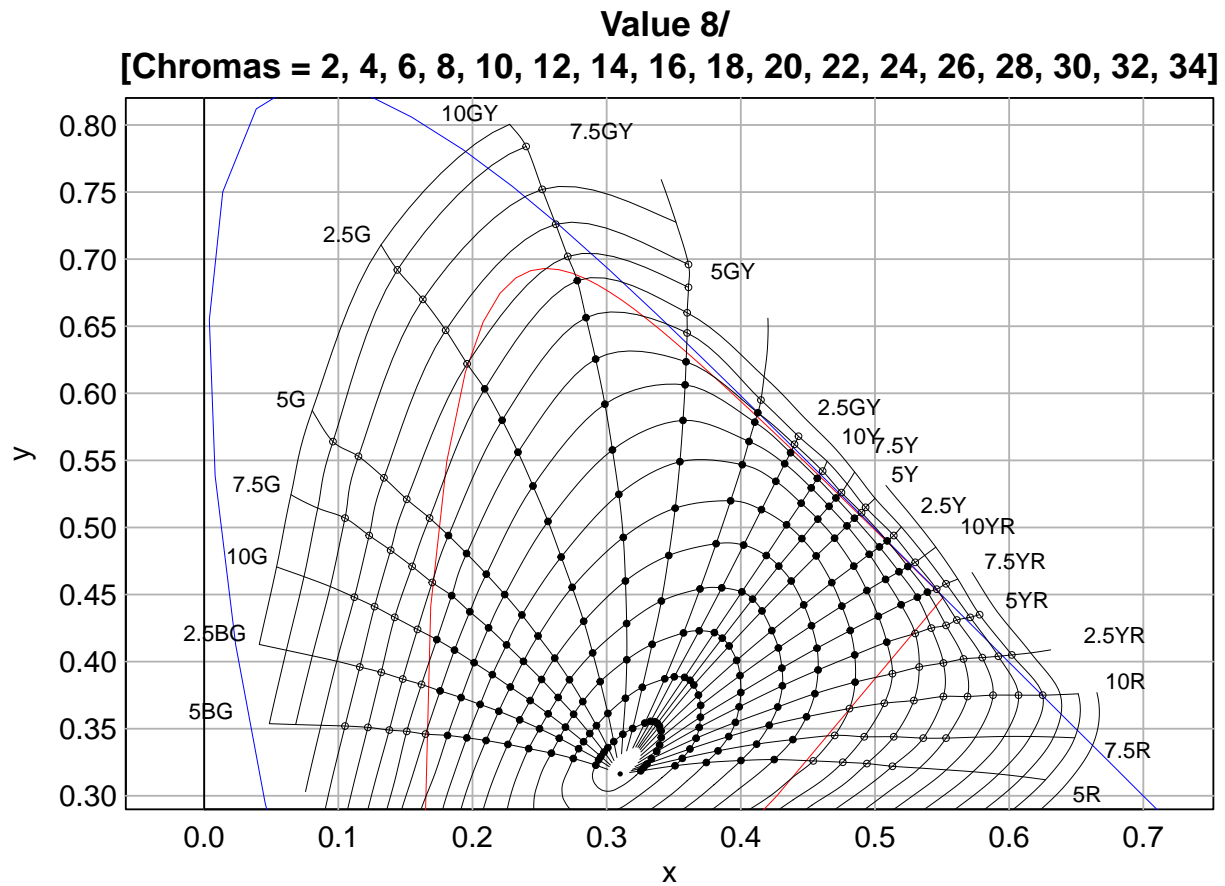
Value 8/
[Chromas = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34]

Figure 2: Loci of Constant Hue and Chroma for Value 8/

And so we see that `all.dat` goes up to C=32 for 10GY, 2.5G, and 5G.

Zoom In

For inversion, the region that gave me the most grief was V < 1 and H near 5P (85). Here is a sample

```
par( omi=c(0,0,0,0), mai=c(0.41,0.7,0.6,0.2) )
plotLociHC( value=0.4, hue=seq(80,90,by=2.5), chroma=seq(2,12,by=2) )
```

Value 0.4/
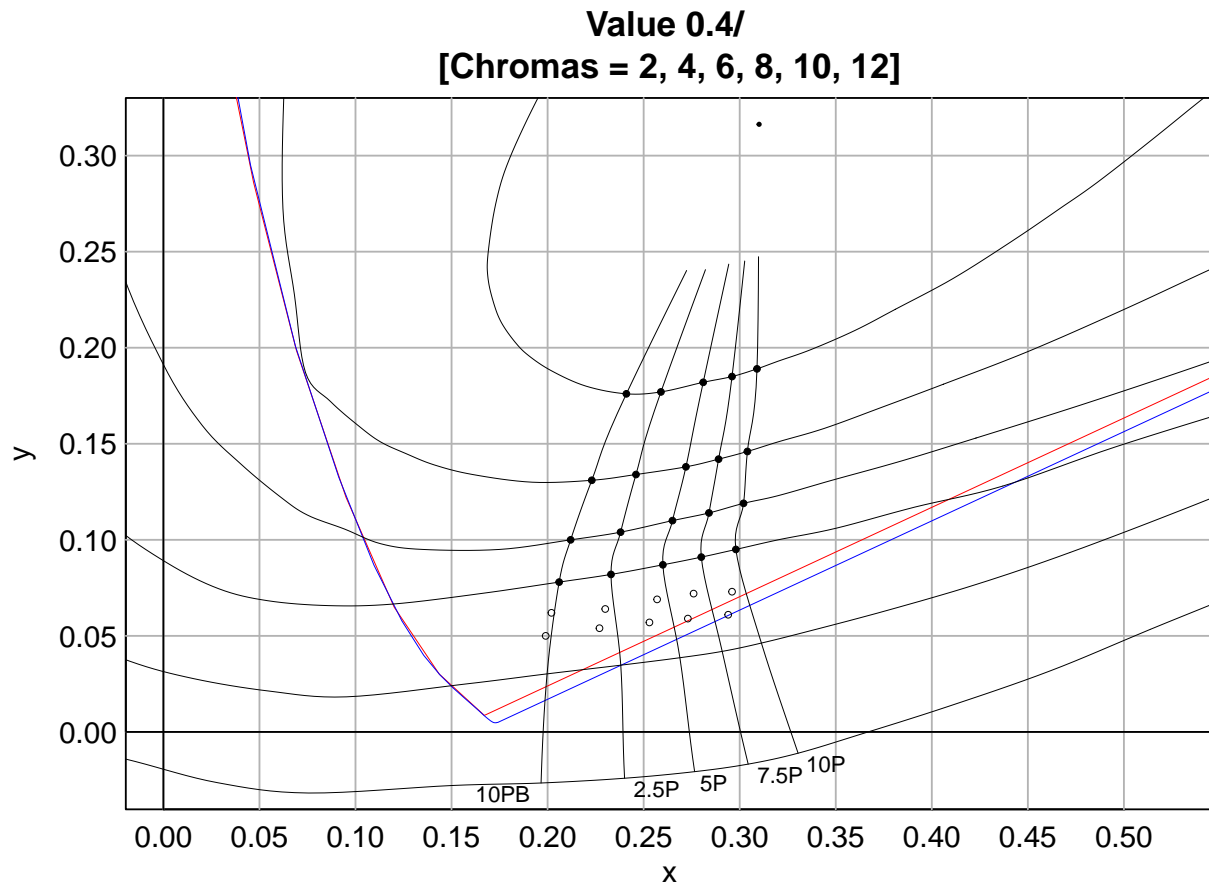**[Chromas = 2, 4, 6, 8, 10, 12]**

Figure 3: Loci of Constant Hue and Chroma for Value 0.4/

It is clear that the curves are not passing through the points from `all.dat`. This is by design, because these points on the opposite side of the hue circle produced badly injective forward maps using cubic interpolation in V. The gridlines for non-real points with V<1 were extrapolated using a thin-plate spline with the function `mgcv::gam()`. It is also clear that this spline is not extraplating lines of constant hue into straight lines - they bend. Finding a better way to handle extrapolation with V<1 is an area for a future version of the package.

Bilinear vs Bicubic Interpolation

In the function `MunsellToxyY()` the default interpolation method for Hue and Chroma is bicubic. This interpolation method computes a weighted average of points on a 4x4 grid. The alternative is bilinear, which only requires a 2x2 grid. The difference in "flops" is roughly a factor of 4 (the difference in time only about 2 or 3) - but is that really worth it ? Here is the original plot, except using bilinear interpolation

```
par( omi=c(0,0,0,0), mai=c(0.41,0.7,0.6,0.2) )
plotLociHC( value=8, hcinterp='bilinear' )
```
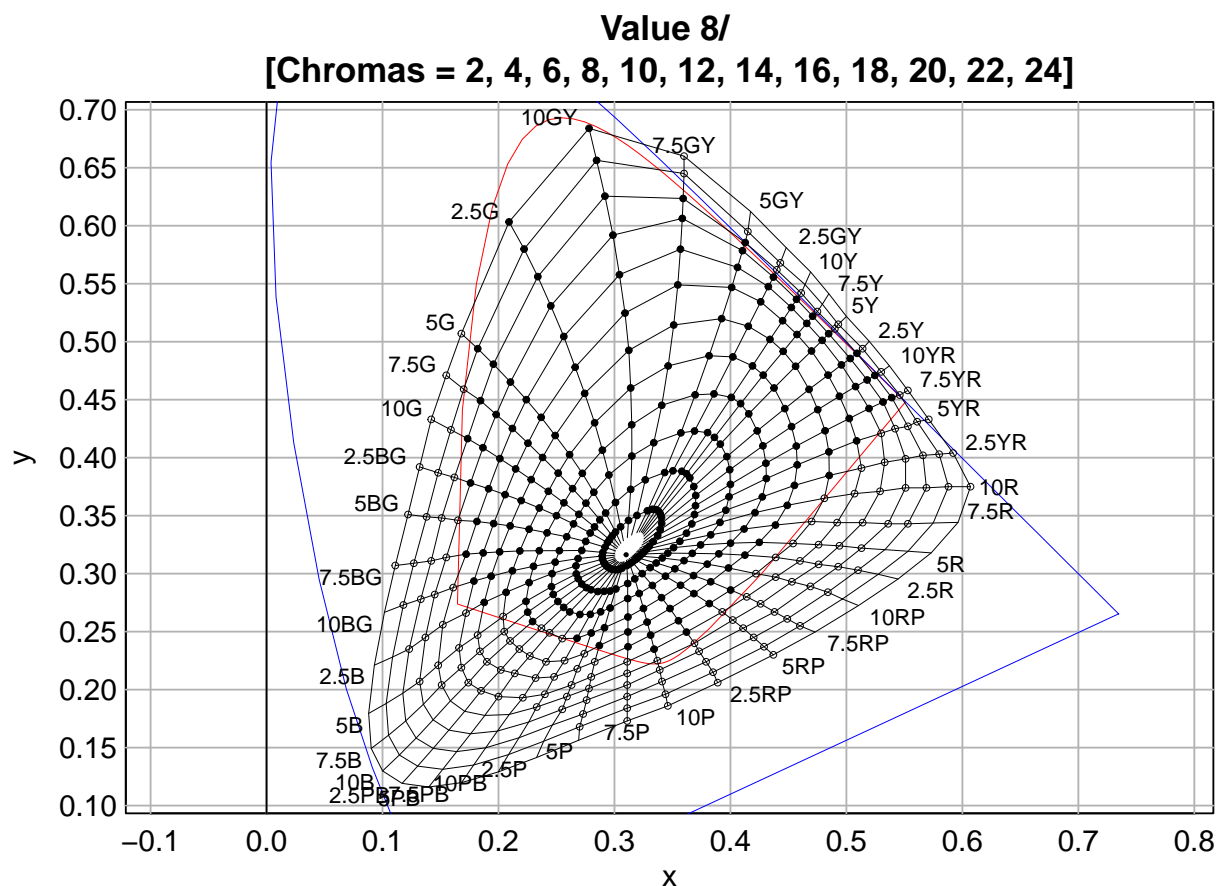
Figure 4: Loci of Constant Hue and Chroma for Value 8/, with bilinear interpolation

Most of this plot is similar to the first one, but in the high curvature region near 10GY the lines are perfectly straight so the plot looks like a spider web. It is certainly not C^1 and does not match Dorothy Nickerson's plots well, so 'yes' I think that bicubic is worth it.

References

[Newhall et al., 1943] Newhall, S. M., Nickerson, D., and Judd, D. B. (1943). Final Report of the O.S.A. Subcommittee on the Spacing of the Munsell Colors. *J. Opt. Soc. Am.*, 33(7):385–418.

[Schleter et al., 1958] Schleter, J. C., Judd, D., and Keegan, H. J. (1958). Extension of the Munsell Renotation System (Abstract). *J. Opt. Soc. Am.*, 48(11):863–864. presented at the Forty-Third Annual Meeting of the Optical Society of America. Statler Hilton Hotel, Detroit, Michigan. October 9, 10, and 11, 1958.

[Wyszecki and Stiles, 1982] Wyszecki, G. and Stiles, W. (1982). *Color Science: Concepts and Methods, Quantitative Data and Formulae, Second Edition.* Wiley Classics Library. John Wiley.

Appendix

This document was prepared June 17, 2025 with the following configuration:

- R version 4.5.1 (2025-06-13 ucrt), `x86_64-w64-mingw32`

- Running under: `Windows 11 x64 (build 26100)`

- Matrix products: default

- Base packages: base, datasets, grDevices, graphics, methods, stats, utils

- Other packages: flextable 0.9.7, knitr 1.50, munsellinterpol 3.2-0, spacesRGB 1.7-0, spacesXYZ 1.6-0

- Loaded via a namespace (and not attached): OxSR 1.0.1, R6 2.6.1, Rcpp 1.0.14, V8 6.0.3, askpass 1.2.1, bslib 0.9.0, cachem 1.1.0, cli 3.6.5, colorSpec 1.8-0, compiler 4.5.1, curl 6.2.2, data.table 1.17.2, digest 0.6.37, equatags 0.2.1, evaluate 1.0.3, fastmap 1.2.0, fontBitstreamVera 0.1.1, fontLiberation 0.1.0, fontquiver 0.2.1, gdtools 0.4.2, glue 1.8.0, grid 4.5.1, highr 0.11, htmltools 0.5.8.1, jquerylib 0.1.4, jsonlite 2.0.0, katex 1.5.0, lifecycle 1.0.4, logger 0.4.0, microbenchmark 1.5.0, officer 0.6.8, openssl 2.3.2, ragg 1.4.0, rlang 1.1.6, rmarkdown 2.29, rootSolve 1.8.2.4, sass 0.4.10, systemfonts 1.2.3, textshaping 1.0.1, tools 4.5.1, uuid 1.2-1, xfun 0.52, xml2 1.3.8, xslt 1.5.1, yaml 2.3.10, zip 2.3.3