Joachim Faulhaber

An Introduction to the Interval Template Library

Lecture
held at the Boost Library Conference 2009

2009-05-08

Updated version 3.2.0   2009-12-02

Slide Design by Chih-Hao Tsai
http://www.chtsai.org

- Background and Motivation

- Design

- Examples

- Semantics

- Implementation

- Future Works

- Availability

- Interval containers simplified the implementation of date and time related tasks
  - Decomposing *"histories"* of attributed events into segments with constant attributes.
  - Working with time grids, e.g. a grid of months.
  - Aggregations of values associated to date or time intervals.

- … that occurred frequently in programs like
  - Billing modules
  - Therapy scheduling programs
  - Hospital and controlling statistics

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- Background is the date time problem domain ...
- … but the scope of the **Itl** as a generic library is more general:

an **interval_set** *is a* **set**
> *that is implemented as a set of intervals*

an **interval_map** *is a* **map**
> *that is implemented as a map of interval value pairs*

- There are two aspects in the design of interval containers
- Fundamental aspect

```cpp
interval_set<int> mySet;
mySet.insert(42);
bool has_answer = mySet.contains(42);
```

  - On the fundamental aspect an interval_set can be used just as a set of elements
  - Set theoretic operations are supported
  - Abstracts from sequential and segmental information
- Segmental aspect
  - Allows to access and iterate over the *segments* of interval containers
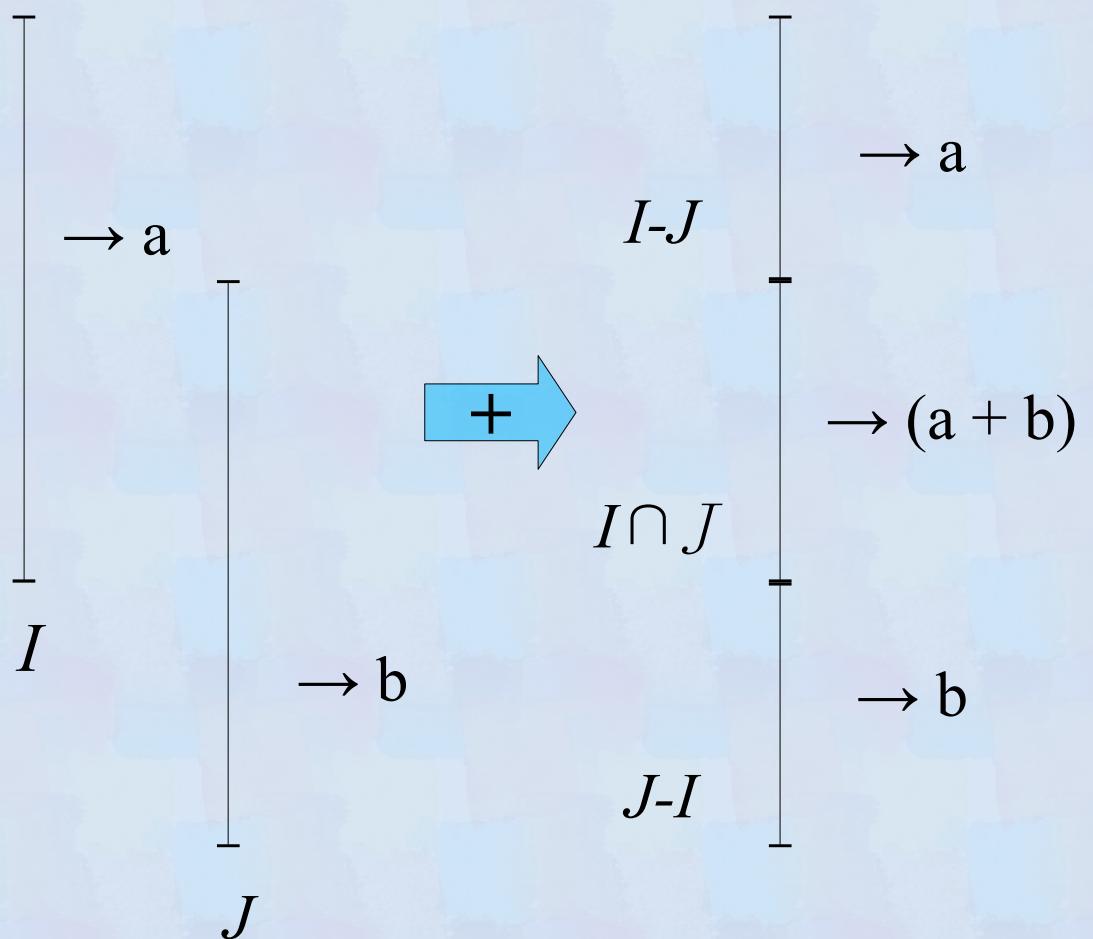
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- Addability and Subtractability
  - All of itl's (interval) containers are *Addable* and *Subtractable*
  - They implement `operators +=, +, -=` and `-`

	`+=`	`-=`
sets	set union	set difference
maps	?	?

- A possible implementation for maps
  - Propagate addition/subtraction to the associated values
  - . . . or aggregate on overlap
  - . . . or aggregate on collision

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- Aggregate on overlap

- Decompositional effect on Intervals

- Accumulative effect on associated values

$\rightarrow a$

I-J $\rightarrow a$

$\rightarrow a$

$+$

$\rightarrow (a + b)$

$I \cap J$

I

$\rightarrow b$ $\rightarrow b$

J-I

J

I, J: intervals, a,b: associated values

- Aggregate on overlap, a minimal example

```
typedef itl::set<string> guests;
interval_map<time, guests> party;

party += make_pair(
  interval<time>::rightopen(20:00, 22:00), guests("Mary"));

party += make_pair(
  interval<time>::rightopen(21:00, 23:00), guests("Harry"));


// party now contains
[20:00, 21:00)->{"Mary"}
[21:00, 22:00)->{"Harry","Mary"} //guest sets aggregated
[22:00, 23:00)->{"Harry"}
```

🔴 The Itl's class templates

Granu-larity	Style	Sets	Maps
interval		`interval`	
	joining	`interval_set`	`interval_map`
	separating	`separate_interval_set`	
	splitting	`split_interval_set`	`split_interval_map`
element		`set`	`map`

- Interval Combining Styles: *Joining*
  - Intervals are joined on overlap or on touch
  - . . . *for maps*, if associated values are equal
  - Keeps interval_maps and sets in a minimal form

```
interval_set

   {[1       3)              }
+         [2        4)
+                    [4 5)

= {[1               4)       }

= {[1                    5)}}
```

```
interval_map

    {[1       3)->1              }
+          [2        4)->1
+                     [4 5)->1

={[1 2)[2 3)[3 4)             }
    ->1   ->2   ->1
={[1 2)[2 3)[3        5)      }
    ->1   ->2        ->1
```

- Interval Combining Styles: *Splitting*
  - Intervals are split on overlap and kept separate on touch
  - All interval borders are preserved (insertion memory)

```
split_interval_set

  {[1        3)             }
+       [2        4)
+                   [4 5)


= {[1 2)[2 3)[3 4)      }

= {[1 2)[2 3)[3 4)[4 5)}
```

```
split_interval_map

  {[1        3)->1            }
+       [2        4)->1
+                   [4 5)->1


={[1 2)[2 3)[3 4)          }
    ->1   ->2   ->1
={[1 2)[2 3)[3 4)[4 5)    }
    ->1   ->2   ->1   ->1
```

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- Interval Combining Styles: *Separating*
  - Intervals are joined on overlap but kept separate on touch
  - Preserves borders that are never crossed (preserves a hidden grid).

```
separate_interval_set

    {[1        3)                }
+            [2        4)
+                        [4 5)

= {[1                 4)        }

= {[1                 4)[4 5)}
```

- A few instances of intervals (interval.cpp)

```cpp
interval<int> int_interval = interval<int>::closed(3,7);

interval<double> sqrt_interval
  = interval<double>::rightopen(1/sqrt(2.0), sqrt(2.0));

interval<std::string> city_interval
  = interval<std::string>::leftopen("Barcelona", "Boston");

interval<boost::ptime> time_interval
  = interval<boost::ptime>::open(
                   time_from_string("2008-05-20 19:30"),
                   time_from_string("2008-05-20 23:00")
    );
```

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🔴 A way to iterate over months and weeks
(`month_and_week_grid.cpp`)

```cpp
#include <boost/itl/gregorian.hpp> //boost::gregorian plus adapter code
#include <boost/itl/split_interval_set.hpp>

// A split_interval_set of gregorian dates as date_grid.
typedef split_interval_set<boost::gregorian::date> date_grid;

// Compute a date_grid of months using boost::gregorian.
date_grid month_grid(const interval<date>& scope)
{
    date_grid month_grid;
    // Compute a date_grid of months using boost::gregorian.
    . . .
    return month_grid;
}

// Compute a date_grid of weeks using boost::gregorian.
date_grid week_grid(const interval<date>& scope)
{
    date_grid week_grid;
    // Compute a date_grid of weeks using boost::gregorian.
    . . .
    return week_grid;
}
```

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- A way to iterate over months and weeks

```
void month_and_time_grid()
{
    date someday = day_clock::local_day();
    date thenday = someday + months(2);
    interval<date> scope = interval<date>::rightopen(someday, thenday);

    // An intersection of the month and week grids ...
    date_grid month_and_week_grid
        = month_grid(scope) & week_grid(scope);

    // ... allows to iterate months and weeks. Whenever a month
    // or a week changes there is a new interval.
    for(date_grid::iterator it = month_and_week_grid.begin();
        it != month_and_week_grid.end(); it++)
    {   . . .   }

    // We can also intersect the grid into an interval_map to make
    // shure that all intervals are within months and week bounds.
    interval_map<boost::gregorian::date, some_type> accrual;
    compute_some_result(accrual, scope);
    accrual &= month_and_week_grid;

}
```

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- Aggregating with interval_maps
  - Computing averages via implementing **operator +=**
    (partys_guest_average.cpp)

```cpp
class counted_sum
{
public:
    counted_sum():_sum(0),_count(0){}
    counted_sum(int sum):_sum(sum),_count(1){}

    int sum()const  {return _sum;}
    int count()const{return _count;}
    double average()const
    { return _count==0 ? 0.0 : _sum/static_cast<double>(_count); }

    counted_sum& operator += (const counted_sum& right)
    { _sum += right.sum(); _count += right.count(); return *this; }

private:
    int _sum;
    int _count;
};

bool operator == (const counted_sum& left, const counted_sum& right)
{ return left.sum()==right.sum() && left.count()==right.count(); }
```

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- Aggregating with interval_maps
  - Computing averages via implementing **operator +=**

```cpp
void partys_height_average()
{
    interval_map<ptime, counted_sum> height_sums;

    height_sums += (
      make_pair(
        interval<ptime>::rightopen(
          time_from_string("2008-05-20 19:30"),
          time_from_string("2008-05-20 23:00")),
          counted_sum(165)) // Mary is 1,65 m tall.
    );

    // Add height of more pary guests . . .

    interval_map<ptime, counted_sum>::iterator height_sum_ =
        height_sums.begin();
    while(height_sum_ != height_sums.end())
    {
        interval<ptime> when = height_sum_->first;
        double height_average = (*height_sum_++).second.average();

        cout << "[" << when.first() << " - " << when.upper() << ")"
             << ": " << height_average << " cm" << endl;
    }
}
```

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- Interval containers allow to express a variety of date and time operations in an easy way.
  - Example `man_power.cpp ...`
  - Subtract weekends and holidays from an interval_set
    ```
    worktime -= weekends(scope)
    worktime -= german_reunification_day
    ```
  - Intersect an interval_map with an interval_set
    ```
    claudias_working_hours &= worktime
    ```
  - Subtract and interval_set from an interval map
    ```
    claudias_working_hours -= claudias_absense_times
    ```
  - Adding interval_maps
    ```
    interval_map<date,int> manpower;
    manpower += claudias_working_hours;
    manpower += bodos_working_hours;
    ```

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🔴 Interval_maps can also be intersected
Example **user_groups.cpp**

```cpp
typedef boost::itl::set<string> MemberSetT;
typedef interval_map<date, MemberSetT> MembershipT;

void user_groups()
{
    . . .

    MembershipT med_users;
    // Compute membership of medical staff
    med_users += make_pair(member_interval_1, MemberSetT("Dr.Jekyll"));
    med_users += . . .

    MembershipT admin_users;
    // Compute membership of administation staff
    med_users += make_pair(member_interval_2, MemberSetT("Mr.Hyde"));
    . . .

    MembershipT all_users   = med_users + admin_users;

    MembershipT super_users = med_users & admin_users;
    . . .

}
```

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- The semantics of *itl sets* is based on a concept `itl::Set`
  - `itl::set`, `interval_set`, `split_interval_set` and `separate_interval_set` are models of concept `itl::Set`

```
// Abstract part
empty set:          Set::Set()
subset relation:    bool Set::contained_in(const Set& s2)const
equality:           bool is_element_equal(const Set& s1, const Set& s2)
set union:          Set& operator += (Set& s1, const Set& s2)
                    Set  operator +  (const Set& s1, const Set& s2)
set difference:     Set& operator -= (Set& s1, const Set& s2)
                    Set  operator -  (const Set& s1, const Set& s2)
set intersection:   Set& operator &= (Set& s1, const Set& s2)
                    Set  operator &  (const Set& s1, const Set& s2)

// Part related to sequential ordering
sorting order:      bool operator <  (const Set& s1, const Set& s2)
lexicographical equality:
                    bool operator == (const Set& s1, const Set& s2)
```
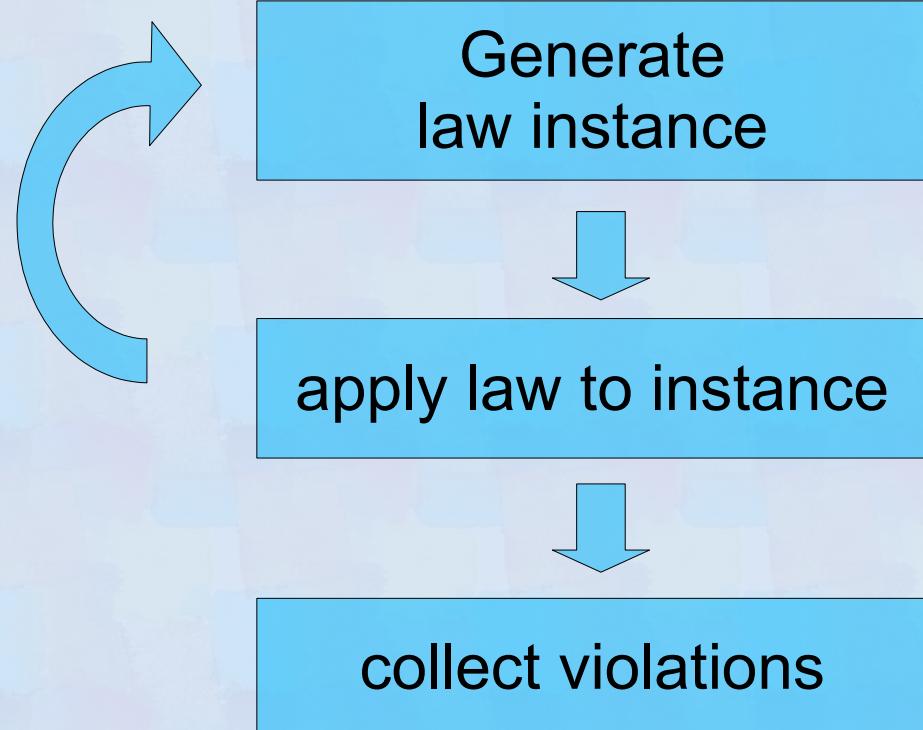
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- The semantics of *itl maps* is based on a concept `itl::Map`
  - `itl::map`, `interval_map` and `split_interval_map` are models of concept `itl::Map`

```
// Abstract part
empty map:          Map::Map()
submap relation:    bool Map::contained_in(const Map& m2)const
equality:           bool is_element_equal(const Map& m1, const Map& m2)
map union:          Map& operator += (Map& m1, const Map& m2)
                    Map  operator +  (const Map& m1, const Map& m2)
map difference:     Map& operator -= (Map& m1, const Map& m2)
                    Map  operator -  (const Map& m1, const Map& m2)
map intersection:   Map& operator &= (Map& m1, const Map& m2)
                    Map  operator &  (const Map& m1, const Map& m2)

// Part related to sequential ordering
sorting order:      bool operator <  (const Map& m1, const Map& m2)
lexicographical equality:
                    bool operator == (const Map& m1, const Map& m2)
```

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- Defining semantics of itl concepts via sets of laws
  - aka c++0x axioms
- Checking law sets via automatic testing:
  - A *La*w *Ba*sed *Te*st *A*utomaton *LaBatea*

Generate
law instance

apply law to instance

collect violations

Commutativity$<$T a, U b, +$>$:
a + b = b + a;

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- Lexicographical Ordering and Equality

  - For all itl containers `operator <` implements a ***strict weak ordering***.

  - The ***induced equivalence*** of this ordering is *lexicographical equality* which is implemented as `operator ==`

  - This is in line with the semantics of SortedAssociativeContainers

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- Subset Ordering and Element Equality

  - For all itl containers function `contained_in` implements a *partial ordering*.

  - The *induced equivalence* of this ordering is *equality of elements* which is implemented as function `is_element_equal`.

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- itl::Sets
- All itl sets implement a ***Set Algebra***, which is to say satisfy a "*classical*" set of laws . . .
  - . . . using `is_element_equal` as equality
  - Associativity, Neutrality, Commutativity (for + and &)
  - Distributivity, DeMorgan, Symmetric Difference

- Most of the itl sets satisfy the classical set of laws even if . . .
  - . . . lexicographical equality: `operator ==` is used
  - The differences reflect proper inequalities in sequence that occur for `separate_interval_set` and `split_interval_set`.

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- Concept Induction / Concept Transition
  - The semantics of itl::Maps appears to be *determined* by the *codomain type* of the map

	is model of	if	example
`Map<D,Monoid>`	`Monoid`		`interval_map<int,string>`
`Map<D,Set>`	`Set`	`C1`	`interval_map<int,set<int>>`
`Map<D,CommutMonoid>`	`CommutMonoid`		`interval_map<int,unsigned>`
`Map<D,AbelianGroup>`	`AbelianGroup`	`C2`	`interval_map<int,int,total>`

  - Conditions *C1* and *C2* restrict the *Concept Induction* to specific *map traits*
    - *C1*: Value pairs that carry a *neutral element* as associated value are **always deleted** (Trait: `absorbs_neutrons`).
    - *C2*: The map *is total*: Non existing keys are implicitly mapped to *neutral elements* (Trait: `is_total`).

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- Itl containers are implemented based on std::set and std::map

  - Basic operations like *adding* and *subtracting* intervals or interval value pairs perform with a time *complexity between* *amortized O(log n)* and *O(n)*, where *n* is the number of intervals of a container.

  - Operations like *addition* and *subtraction* of whole containers are having a worst case complexity of *O(m log(n+m)),* where *n* and *m* are the numbers of intervals of the containers to combine.

    * : Consult the library documentation for more detailed information.

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- Implementing interval_maps of sets more efficiently

- Revision of features of the extended itl (itl_plus.zip)
    - Decomposition of histories: k histories h_k with attribute types A_1, ..., A_k are "*decomposed*" to a product history of tuples of attribute sets:
    $(h_1<T,A_1>,..., h<T,A_k>) \rightarrow h<T, (set<A_1>,…, set<A_k>)>$

    - Cubes (generalized crosstables): Applying *aggregate on collision* to maps of tuple value pairs in order to organize hierachical data and their aggregates.

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- Itl project on sourceforge (version 2.0.1)
  http://sourceforge.net/projects/itl

- Latest version on boost vault/Containers (3.2.0)
  http://www.boostpro.com/vault/ → containers

  - itl_3_2_0.zip : Core itl in preparation for boost

  - itl_plus_3_2_0.zip : Extended itl including histories, cubes and automatic validation (LaBatea).

- Online documentation at
  http://www.herold-faulhaber.de/

  - Doxygen generated docs for (version 2.0.1)
    http://www.herold-faulhaber.de/itl/

  - Latest boost style documentation (version 3.2.0)
    http://www.herold-faulhaber.de/boost_itl/doc/libs/itl/doc/html/

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- Boost sandbox
  https://svn.boost.org/svn/boost/sandbox/itl/

  - Core itl: Interval containers proposed for boost
    https://svn.boost.org/svn/boost/sandbox/itl/boost/itl/
    https://svn.boost.org/svn/boost/sandbox/itl/libs/itl/

  - Extended itl_xt: interval_bitset, "histories", cubes
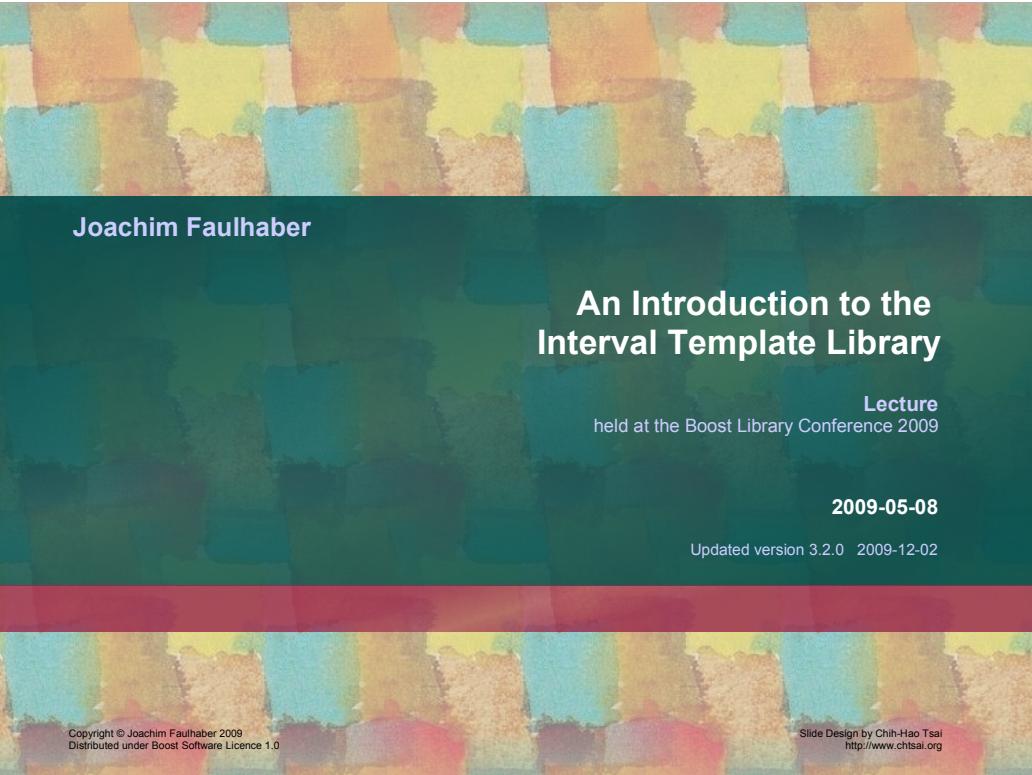   https://svn.boost.org/svn/boost/sandbox/itl/boost/itl_xt/
    https://svn.boost.org/svn/boost/sandbox/itl/libs/itl_xt/

  - Validater LaBatea:
    Compiles with msvc-8.0 or newer, gcc-4.3.2 or newer
    https://svn.boost.org/svn/boost/sandbox/itl/boost/validate/
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bool has_answer = mySet.contains(42);
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- Segmental aspect
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- Addability and Subtractability
  - All of itl's (interval) containers are *Addable* and *Subtractable*
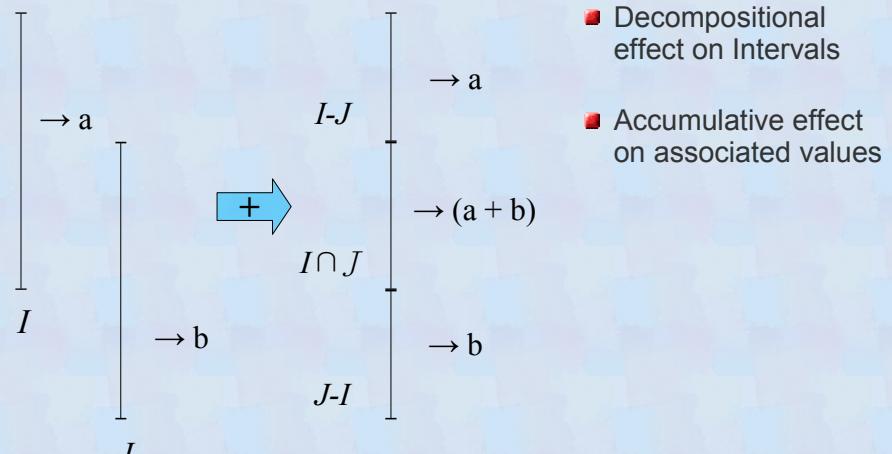 - They implement **operators +=, +, -=** and **-**

	+=	**-=**
sets	set union	set difference
maps	?	?

- A possible implementation for maps
  - Propagate addition/subtraction to the associated values
  - . . . or aggregate on overlap
  - . . . or aggregate on collision

6

● Aggregate on overlap

$\to a$

I

$\to b$

J

$+$

$I\text{-}J$ $\to a$

$I \cap J$ $\to (a + b)$

$J\text{-}I$ $\to b$

■ Decompositional effect on Intervals

■ Accumulative effect on associated values

I, J: intervals, a,b: associated values

● Aggregate on overlap, a minimal example

```cpp
typedef itl::set<string> guests;
interval_map<time, guests> party;

party += make_pair(
  interval<time>::rightopen(20:00, 22:00), guests("Mary"));

party += make_pair(
  interval<time>::rightopen(21:00, 23:00), guests("Harry"));


// party now contains
[20:00, 21:00)->{"Mary"}
[21:00, 22:00)->{"Harry","Mary"} //guest sets aggregated
[22:00, 23:00)->{"Harry"}
```

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🔴 The Itl's class templates

Granu-larity	Style	Sets	Maps
interval		`interval`	
	joining	`interval_set`	`interval_map`
	separating	`separate_interval_set`	
	splitting	`split_interval_set`	`split_interval_map`
element		`set`	`map`

● Interval Combining Styles: *Joining*

   ■ Intervals are joined on overlap or on touch

   ■ . . . *for maps*, if associated values are equal

   ■ Keeps interval_maps and sets in a minimal form

```
interval_set                    interval_map

  {[1        3)            }       {[1        3)->1          }
+        [2       4)        +              [2        4)->1
+                  [4 5)    +                      [4 5)->1

= {[1            4)      }      ={[1 2)[2 3)[3 4)         }
                                   ->1   ->2   ->1
= {[1                5)}        ={[1 2)[2 3)[3        5)   }
                                   ->1   ->2        ->1
```

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- Interval Combining Styles: *Splitting*
  - Intervals are split on overlap and kept separate on touch
  - All interval borders are preserved (insertion memory)

```
split_interval_set

  {[1       3)            }
+      [2        4)
+                   [4 5)

= {[1 2)[2 3)[3 4)        }

= {[1 2)[2 3)[3 4)[4 5)}
```

```
split_interval_map

  {[1       3)->1           }
+      [2        4)->1
+                   [4 5)->1

={[1 2)[2 3)[3 4)         }
    ->1  ->2   ->1
={[1 2)[2 3)[3 4)[4 5)   }
    ->1  ->2   ->1   ->1
```

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- Interval Combining Styles: *Separating*
  - Intervals are joined on overlap but kept separate on touch
  - Preserves borders that are never crossed (preserves a hidden grid).

```
separate_interval_set

  {[1        3)              }
+         [2        4)
+                   [4 5)

= {[1              4)       }

= {[1              4)[4 5)}
```

● A few instances of intervals (interval.cpp)

```cpp
interval<int> int_interval = interval<int>::closed(3,7);

interval<double> sqrt_interval
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(`month_and_week_grid.cpp`)

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#include <boost/itl/split_interval_set.hpp>

// A split_interval_set of gregorian dates as date_grid.
typedef split_interval_set<boost::gregorian::date> date_grid;

// Compute a date_grid of months using boost::gregorian.
date_grid month_grid(const interval<date>& scope)
{
    date_grid month_grid;
    // Compute a date_grid of months using boost::gregorian.
    . . .
    return month_grid;
}

// Compute a date_grid of weeks using boost::gregorian.
date_grid week_grid(const interval<date>& scope)
{
    date_grid week_grid;
    // Compute a date_grid of weeks using boost::gregorian.
    . . .
    return week_grid;
}
```

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```cpp
void month_and_time_grid()
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    date someday = day_clock::local_day();
    date thenday = someday + months(2);
    interval<date> scope = interval<date>::rightopen(someday, thenday);

    // An intersection of the month and week grids ...
    date_grid month_and_week_grid
        = month_grid(scope) & week_grid(scope);

    // ... allows to iterate months and weeks. Whenever a month
    // or a week changes there is a new interval.
    for(date_grid::iterator it = month_and_week_grid.begin();
        it != month_and_week_grid.end(); it++)
    {   . . .   }

    // We can also intersect the grid into an interval_map to make
    // shure that all intervals are within months and week bounds.
    interval_map<boost::gregorian::date, some_type> accrual;
    compute_some_result(accrual, scope);
    accrual &= month_and_week_grid;

}
```

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  - Computing averages via implementing **operator** **+=**
    (partys_guest_average.cpp)

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{
public:
    counted_sum(): _sum(0), _count(0){}
    counted_sum(int sum): _sum(sum), _count(1){}

    int sum()const   {return _sum;}
    int count()const{return _count;}
    double average()const
    { return _count==0 ? 0.0 : _sum/static_cast<double>(_count); }

    counted_sum& operator += (const counted_sum& right)
    { _sum += right.sum(); _count += right.count(); return *this; }

private:
    int _sum;
    int _count;
};

bool operator == (const counted_sum& left, const counted_sum& right)
{ return left.sum()==right.sum() && left.count()==right.count(); }
```

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- Aggregating with interval_maps
  - Computing averages via implementing **operator** **+=**

```
void partys_height_average()
{
    interval_map<ptime, counted_sum> height_sums;

    height_sums += (
      make_pair(
        interval<ptime>::rightopen(
          time_from_string("2008-05-20 19:30"),
          time_from_string("2008-05-20 23:00")),
        counted_sum(165)) // Mary is 1,65 m tall.
    );

    // Add height of more pary guests . . .

    interval_map<ptime, counted_sum>::iterator height_sum_ =
        height_sums.begin();
    while(height_sum_ != height_sums.end())
    {
        interval<ptime> when = height_sum_->first;
        double height_average = (*height_sum_++).second.average();

        cout << "[" << when.first() << " - " << when.upper() << ")"
             << ": " << height_average << " cm" << endl;
    }
}
```

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- Interval containers allow to express a variety of date and time operations in an easy way.
  - Example `man_power.cpp ...`
  - Subtract weekends and holidays from an interval_set
    ```
    worktime -= weekends(scope)
    worktime -= german_reunification_day
    ```
  - Intersect an interval_map with an interval_set
    ```
    claudias_working_hours &= worktime
    ```
  - Subtract and interval_set from an interval map
    ```
    claudias_working_hours -= claudias_absense_times
    ```
  - Adding interval_maps
    ```
    interval_map<date,int> manpower;
    manpower += claudias_working_hours;
    manpower += bodos_working_hours;
    ```

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- Interval_maps can also be intersected
  Example **user_groups.cpp**

```cpp
typedef boost::itl::set<string> MemberSetT;
typedef interval_map<date, MemberSetT> MembershipT;

void user_groups()
{
    . . .

    MembershipT med_users;
    // Compute membership of medical staff
    med_users += make_pair(member_interval_1, MemberSetT("Dr.Jekyll"));
    med_users += . . .

    MembershipT admin_users;
    // Compute membership of administation staff
    med_users += make_pair(member_interval_2, MemberSetT("Mr.Hyde"));
    . . .

    MembershipT all_users   = med_users + admin_users;

    MembershipT super_users = med_users & admin_users;
    . . .

}
```

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● The semantics of *itl sets* is based on a concept `itl::Set`

■ `itl::set`, `interval_set`, `split_interval_set` and `separate_interval_set` are models of concept `itl::Set`

```
// Abstract part
empty set:        Set::Set()
subset relation:  bool Set::contained_in(const Set& s2)const
equality:         bool is_element_equal(const Set& s1, const Set& s2)
set union:        Set& operator += (Set& s1, const Set& s2)
                  Set  operator +  (const Set& s1, const Set& s2)
set difference:   Set& operator -= (Set& s1, const Set& s2)
                  Set  operator -  (const Set& s1, const Set& s2)
set intersection: Set& operator &= (Set& s1, const Set& s2)
                  Set  operator &  (const Set& s1, const Set& s2)

// Part related to sequential ordering
sorting order:    bool operator < (const Set& s1, const Set& s2)
lexicographical equality:
                  bool operator == (const Set& s1, const Set& s2)
```
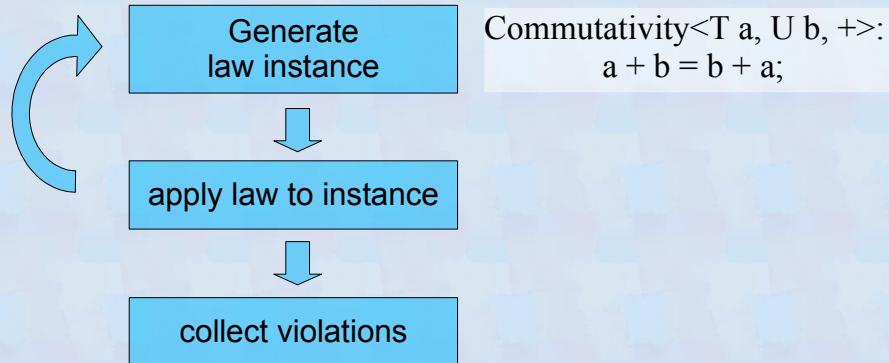
20

- The semantics of *itl maps* is based on a concept `itl::Map`

  - `itl::map`, `interval_map` and `split_interval_map` are models of concept
    `itl::Map`

```
// Abstract part
empty map:         Map::Map()
submap relation:   bool Map::contained_in(const Map& m2)const
equality:          bool is_element_equal(const Map& m1, const Map& m2)
map union:         Map& operator += (Map& m1, const Map& m2)
                   Map  operator +  (const Map& m1, const Map& m2)
map difference:    Map& operator -= (Map& m1, const Map& m2)
                   Map  operator -  (const Map& m1, const Map& m2)
map intersection:  Map& operator &= (Map& m1, const Map& m2)
                   Map  operator &  (const Map& m1, const Map& m2)

// Part related to sequential ordering
sorting order:     bool operator <  (const Map& m1, const Map& m2)
lexicographical equality:
                   bool operator == (const Map& m1, const Map& m2)
```

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- Defining semantics of itl concepts via sets of laws
  - aka c++0x axioms
- Checking law sets via automatic testing:
  - A *La*w *Ba*sed *Te*st *A*utomaton *LaBatea*

Generate law instance

Commutativity$<$T a, U b, +$>$:
$$a + b = b + a;$$

apply law to instance

collect violations

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- Lexicographical Ordering and Equality

  - For all itl containers `operator <` implements a ***strict weak ordering***.

  - The ***induced equivalence*** of this ordering is ***lexicographical equality*** which is implemented as `operator ==`

  - This is in line with the semantics of SortedAssociativeContainers

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- Subset Ordering and Element Equality

  - For all itl containers function `contained_in` implements a *partial ordering*.

  - The *induced equivalence* of this ordering is *equality of elements* which is implemented as function `is_element_equal`.

- itl::Sets
- **All** itl sets implement a ***Set Algebra***, which is to say satisfy a "*classical*" set of laws . . .
  - . . . using `is_element_equal` as equality
  - Associativity, Neutrality, Commutativity (for **+** and **&**)
  - Distributivity, DeMorgan, Symmetric Difference

- **Most of** the itl sets satisfy the classical set of laws even if . . .
  - . . . lexicographical equality: `operator ==` is used
  - The differences reflect proper inequalities in sequence that occur for `separate_interval_set` and `split_interval_set`.

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🔴 Concept Induction / Concept Transition

   🔺 The semantics of itl::Maps appears to be *determined* by the *codomain type* of the map

	is model of	if	example
`Map<D,Monoid>`	Monoid		`interval_map<int,string>`
`Map<D,Set>`	Set	*C1*	`interval_map<int,set<int>>`
`Map<D,CommutMonoid>`	CommutMonoid		`interval_map<int,unsigned>`
`Map<D,AbelianGroup>`	AbelianGroup	*C2*	`interval_map<int,int,total>`

   🔺 Conditions *C1* and *C2* restrict the *Concept Induction* to specific *map traits*

      🔵 *C1*: Value pairs that carry a *neutral element* as associated value are **always deleted** (Trait: ***absorbs_neutrons***).

      🔵 *C2*: The map *is total*: Non existing keys are implicitly mapped to *neutral elements* (Trait: ***is_total***).

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- Itl containers are implemented based on std::set and std::map

    - Basic operations like *adding* and *subtracting* intervals or interval value pairs perform with a time *complexity between* * *amortized O(log n)* and *O(n)*, where *n* is the number of intervals of a container.

    - Operations like *addition* and *subtraction* of whole containers are having a worst case complexity of *O(m log(n+m)),* where *n* and *m* are the numbers of intervals of the containers to combine.

        * : Consult the library documentation for more detailed information.

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- Implementing interval_maps of sets more efficiently

- Revision of features of the extended itl (itl_plus.zip)
  - Decomposition of histories: k histories h_k with attribute types $A_1, ..., A_k$ are "*decomposed*" to a product history of tuples of attribute sets:
  $(h_1\langle T,A_1\rangle,..., h\langle T,A_k\rangle) \rightarrow h\langle T, (\text{set}\langle A_1\rangle,..., \text{set}\langle A_k\rangle)\rangle$

  - Cubes (generalized crosstables): Applying *aggregate on collision* to maps of tuple value pairs in order to organize hierachical data and their aggregates.

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- Itl project on sourceforge (version 2.0.1)
  http://sourceforge.net/projects/itl
- Latest version on boost vault/Containers (3.2.0)
  http://www.boostpro.com/vault/ → containers
  - itl_3_2_0.zip : Core itl in preparation for boost
  - itl_plus_3_2_0.zip : Extended itl including histories, cubes and automatic validation (LaBatea).
- Online documentation at
  http://www.herold-faulhaber.de/
  - Doxygen generated docs for (version 2.0.1)
    http://www.herold-faulhaber.de/itl/
  - Latest boost style documentation (version 3.2.0)
    http://www.herold-faulhaber.de/boost_itl/doc/libs/itl/doc/html/

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- Boost sandbox
  https://svn.boost.org/svn/boost/sandbox/itl/

  - Core itl: Interval containers proposed for boost
    https://svn.boost.org/svn/boost/sandbox/itl/boost/itl/
    https://svn.boost.org/svn/boost/sandbox/itl/libs/itl/
  - Extended itl_xt: interval_bitset, "histories", cubes
    https://svn.boost.org/svn/boost/sandbox/itl/boost/itl_xt/
    https://svn.boost.org/svn/boost/sandbox/itl/libs/itl_xt/
  - Validater LaBatea:
    Compiles with msvc-8.0 or newer, gcc-4.3.2 or newer
    https://svn.boost.org/svn/boost/sandbox/itl/boost/validate/
    https://svn.boost.org/svn/boost/sandbox/itl/libs/validate/

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